

# Electron Capture in a Fully Ionized Plasma

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Properties of fully ionized water plasmas are discussed including plasma charge density oscillations and the screening of the Coulomb law especially in the dilute classical Debye regime. A kinetic model with two charged particle scattering events determines the transition rate per unit time for electron capture by a nucleus with the resulting nuclear transmutations. Two corrections to the recent Maiani *et.al.* calculations are made: (i) The Debye screening length is only employed within its proper domain of validity. (ii) The WKB approximation employed by Maiani in the long De Broglie wave length limit is evidently invalid. We replace this incorrect approximation with mathematically rigorous Calogero inequalities in order to discuss the scattering wave functions. Having made these corrections, we find a verification for our previous results based on condensed matter electro-weak quantum field theory for nuclear transmutations in chemical batteries.

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## I. INTRODUCTION

In recent years we have been working on electro-weak interaction inverse beta decay by including including electro-magnetic interactions with collective plasma modes of motion[1, 2]. We have applied this theory to electron capture in a water plasma to explain observed nuclear transmutations on the cathode surface of a chemical cell[3]. While the original theory was formulated in terms of electro-weak quantum field theory[4] in a many body context, a reasonable alternative relies on physical kinetic plasma theory[5–8] to describe a water plasma. The theoretical kinetic model gives rise to electron capture rates per unit time per unit cathode surface area in a water plasma in agreement with the quantum field theoretical model and is in agreement with experiments.

Objections based on the kinetic model in a *cold plasma* were raised by the Rome group Ciuchi *et.al.*[9]. They find electron capture rates about two orders of magnitude lower than our previous work. The objection was answered by pointing out that the water plasma in a chemical cell can light up the laboratory and thereby represents a *hot plasma*. The *hot fully ionized plasma* gives rise to an increased electron capture rate in agreement with experiment and in agreement with our previous results. This has been previously and fully discussed[10, 11].

Most recently it has been predicted by the Rome group, Maiani *et.al.*[12], that a *cold plasma* has a higher rate of electron capture than does a *hot plasma*. This has been predicted by Maiani on the basis of (i) the Debye screening of the attractive Coulomb interactions between the electron and the proton and (ii) by the quasi-classical WKB approximation to the s-wave electron-proton wave function. The prediction is in flagrant disagreement with experiments which exhibit a hot water plasma nuclear transmutations and do not exhibit such transmutations in a cold plasma. The Maiani computation fails theoretically because (i) The Debye screening length is applied

in regimes wherein it is clearly invalid and (ii) the WKB approximation is applied in the long De Broglie wave length regime but in reality the WKB approximation is valid only in the short De Broglie wave length regime. One of our purposes is to correct the errors made by Maiani. When the properly rigorous mathematics is applied we recover our previous and correct results.

In Sec.II, rigorous sum rules for the plasma oscillation frequency and the plasma screening length is reviewed. A general thermodynamic expression for the screening length is found in Sec.II A. The Debye screening length is then derived in Sec.II B and the regime of the validity of the Debye theory is clearly specified.

In Sec.III the expression for an electron capture transition rate per unit time per nucleus is derived in terms of the electron nuclear correlation function

$$\bar{n} = \left\langle \sum_j \delta(\mathbf{R} - \mathbf{r}_j) \right\rangle \quad (1)$$

describing the density of electrons at positions  $\{\mathbf{r}_j\}$  sitting right on top of a nucleus at position  $\mathbf{R}$ . The effects of the plasma on electron capture transition rates is described by  $\bar{n}$ . For the case of a water plasma on the cathode surface of a chemical cell exhibiting nuclear transmutations, the length scales are discussed and estimated in Sec.III A. In Sec.III B, a hot Debye screened plasma result is derived for  $\bar{n}$  equivalent to our previous calculations[10, 11] but in disagreement with Maiani *et.al.*[12] for reasons discussed above.

The Rome group in reality calculates[12] for some densities in the quantum degenerate zero temperature regime wherein Thomas-Fermi quantum screening plasma replaces classical Debye screening plasma. In practical terms, this regime requires the solution of the radial s-wave potential scattering equation

$$-\left(\frac{\hbar^2}{2\mu}\right) \frac{d^2 u(r)}{dr^2} + \mathcal{U}(r)u(r) = Eu(r), \quad (2)$$

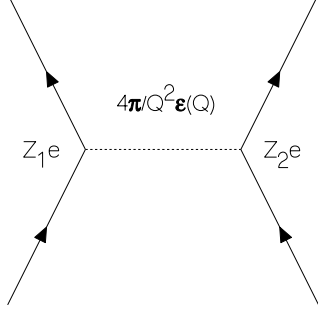


FIG. 1: The interaction  $4\pi e^2 Z_1 Z_2 / Q^2 \varepsilon(Q)$  determines the screened Coulomb interaction potential energy between two charged particles due to one photon exchange. The static interaction potential energy is given in Eq.(3).

wherein  $\mathcal{U}(r)$  is the two charged particle screened potential. In solving the problem for pure s-wave scattering one conventionally takes the limit  $E \rightarrow 0$ . Maiani[12] unconventionally employs the WKB method valid only for  $E \rightarrow \infty$  wherein phase shifts other than s-wave gain importance. These two energy limits are different. Maiani is in error in his calculation of s-wave potential scattering wave functions.

For completeness of presentation, in Sec.IV we review the proper method of computing s-wave scattering wave functions employing the mathematically rigorous variable phase formalism[13] of Calogero. In the limit  $E \rightarrow 0$ , one may compute the scattering wave functions in terms of the variable scattering length as discussed in Sec.IV A. The implications for electron capture rates is discussed in Sec.IV B. In the concluding Sec.V a summary is given of the results of this work.

## II. SCREENING THEORY

The dielectric response of a plasma to external charge distributions is described by a wave number  $\mathbf{Q}$  and complex frequency  $\zeta$  dependent dielectric function  $\varepsilon(Q, \zeta)$  in the upper half frequency plane  $\Im m \zeta > 0$ . The static dielectric response function  $\varepsilon(Q) = \lim_{\omega \rightarrow 0} \varepsilon(Q, \omega + i0^+)$  determines the one photon exchange screened Coulomb interaction between two charges  $Z_1 e$  and  $Z_2 e$ ,

$$\mathcal{U}(r) = Z_1 Z_2 e^2 \int e^{i\mathbf{Q} \cdot \mathbf{r}} \left[ \frac{4\pi}{Q^2 \varepsilon(Q)} \right] \frac{d^3 \mathbf{Q}}{(2\pi)^3},$$

$$\mathcal{U}(r) = \left( \frac{Z_1 Z_2 e^2}{r} \right) \mathcal{S}(r), \quad (3)$$

with a Coulomb law screening function

$$\mathcal{S}(r) = \left( \frac{2}{\pi} \right) \int_0^\infty \sin(Qr) \left[ \frac{dQ}{Q \varepsilon(Q)} \right],$$

$$\frac{1}{Q \varepsilon(Q)} = \int_0^\infty \sin(Qr) S(r) dr, \quad (4)$$

as shown in FIG. 1.

The screening length  $\Lambda = \kappa^{-1}$  may be defined as the equivalent limits

$$\varepsilon(Q) = 1 + \frac{\kappa^2}{Q^2} + \dots \quad (Q \rightarrow 0),$$

$$S(r) \rightarrow e^{-r/\Lambda} + \dots \quad (r \rightarrow \infty). \quad (5)$$

In virtue of Eqs.(4) and (5) one may write the formal limits

$$\kappa^2 = \lim_{Q \rightarrow 0} Q^2 \varepsilon(Q),$$

$$\frac{1}{\kappa^2} = \Lambda^2 = \int_0^\infty r S(r) dr. \quad (6)$$

If we choose the limit  $Q \rightarrow 0$  first, then the plasma conductivity  $\sigma(\zeta)$  is defined and obeys a dispersion relation,

$$\lim_{Q \rightarrow 0} \varepsilon(Q, \zeta) = 1 + \frac{4\pi i \sigma(\zeta)}{\zeta}$$

$$\sigma(\zeta) = - \left( \frac{2i\zeta}{\pi} \right) \int_0^\infty \frac{\Re \sigma(\omega + i0^+) d\omega}{(\omega^2 - \zeta^2)}. \quad (7)$$

The frequency of plasma oscillations  $\omega_p$  may be found from the sum rule

$$4\pi\sigma(\zeta) \rightarrow \frac{i\omega_p^2}{\zeta} + \dots \quad (|\zeta| \rightarrow \infty),$$

$$\left( \frac{2}{\pi} \right) \int_0^\infty \Re \sigma(\omega + i0^+) d\omega = \frac{\omega_p^2}{4\pi},$$

$$\omega_p^2 = 4\pi e^2 \sum_a \left( \frac{z_a^2 n_a}{m_a} \right). \quad (8)$$

In Eq.(8) the plasma component with charge  $z_a e$  and mass  $m_a$  exists with a density of  $n_a$  per unit volume. The thermodynamic velocity  $u_T$  may be defined as

$$u_T = \left( \frac{\omega_p}{\kappa} \right). \quad (9)$$

The zero frequency conductivity  $\sigma$  defines a plasma relaxation time  $\tau$  via  $4\pi\sigma = \omega_p^2 \tau$  or equivalently the charge diffusion coefficient  $D = u_T^2 \tau$  which in virtue of Eq.(9) yields the Einstein relation

$$4\pi\sigma = D \kappa^2. \quad (10)$$

The above results are rigorously true for non-relativistic Coulomb plasma phases of matter. It is important to derive one further thermodynamic sum rule for the screening length  $\Lambda = \kappa^{-1}$ .

### A. Thermodynamic Sum Rules

The thermodynamic pressure  $P(T, \mu_1, \dots, \mu_f)$  completely determines the equations of state of the plasma

$$dP = sdT + \sum_a n_a d\mu_a. \quad (11)$$

Let us consider the charge in a macroscopic subvolume  $V$  and the charge contained within that subvolume. Since the plasma is neutral, the mean charge is zero; i.e.

$$\bar{Q} = e \sum_a z_a \bar{N}_a = 0. \quad (12)$$

There are nevertheless charge fluctuations within the subvolume

$$\begin{aligned} \overline{\Delta Q^2} &= e^2 \sum_{a,b} z_a z_b \overline{\Delta N_a \Delta N_b}, \\ \overline{\Delta Q^2} &= k_B T \left( \frac{\partial \bar{Q}}{\partial \Phi} \right)_T = k_B T C_s, \\ \overline{\Delta N_a \Delta N_b} &= V k_B T \left( \frac{\partial^2 P}{\partial \mu_a \partial \mu_b} \right)_T, \\ \overline{\Delta Q^2} &= V k_B T e^2 \sum_{a,b} z_a z_b \left( \frac{\partial^2 P}{\partial \mu_a \partial \mu_b} \right)_T, \end{aligned} \quad (13)$$

wherein  $\Phi$  is a uniform electrostatic potential, the self capacitance of the volume  $V$  is  $C_s$  and statistical thermodynamic fluctuation theory[14] has been invoked. Evidently,

$$\frac{C_s}{V} = e^2 \sum_{a,b} z_a z_b \left( \frac{\partial^2 P}{\partial \mu_a \partial \mu_b} \right)_T. \quad (14)$$

The energy associated with a uniformly charge macroscopic subvolume  $V$  obeys

$$\begin{aligned} \mathcal{E} &= \frac{1}{2} \int \int \left[ \frac{\rho_1 \rho_2}{r_{12}} \right] \mathcal{S}(r_{12}) d^3 \mathbf{r}_1 d^3 \mathbf{r}_2, \\ \mathcal{E} &= \frac{\Delta Q^2}{2V} \int \frac{\mathcal{S}(r)}{r} d^3 \mathbf{r} = \frac{\Delta Q^2}{2C_s}, \end{aligned} \quad (15)$$

so that

$$\frac{V}{C_s} = 4\pi \int_0^\infty r \mathcal{S}(r) dr = \frac{4\pi}{\kappa^2} = 4\pi \Lambda^2 \quad (16)$$

wherein Eq.(6) has been invoked. In virtue of Eqs.(14) and (16) we have proved the following

**Theorem:** The screening length  $\Lambda = \kappa^{-1}$  is determined by the thermodynamic identity

$$\kappa^2 = 4\pi e^2 \sum_{a,b} z_a z_b \left( \frac{\partial^2 P}{\partial \mu_a \partial \mu_b} \right)_T. \quad (17)$$

Depending on the equations of state implicit in Eq.(11), different screening lengths will appear in different regimes. For example, if the electrons are in a high temperature regime then the classical Debye screening length holds true. If the electrons are in a low temperature degenerate regime then the Thomas-Fermi screening length holds true. Let us consider the Debye screening regime.

### B. Debye Screening

If the charged particles in the plasma are dilutely distributed then the particle number fluctuations

$$\begin{aligned} \overline{\Delta N_a \Delta N_b} &= V k_B T \left( \frac{\partial^2 P}{\partial \mu_a \partial \mu_b} \right)_T, \\ \overline{\Delta N_a \Delta N_b} &= \delta_{ab} \bar{N}_a \quad (\text{dilute charged particles}). \end{aligned} \quad (18)$$

Eqs.(13), (17) and (18) imply the Debye screening length  $\Lambda_D = \kappa_D^{-1}$  is given by

$$\begin{aligned} \frac{1}{\Lambda_D^2} &= \kappa_D^2 = \frac{4\pi e^2 \mathcal{N}}{k_B T} \quad (\text{Debye Screening}), \\ \mathcal{N} &= \sum_a z_a^2 n_a = \frac{1}{L^3} \quad (\text{Ionicity}). \end{aligned} \quad (19)$$

We will consider below electron proton scattering wherein the electron has a *heavy mass*  $\mu$  with an effective Bohr radius

$$a = \frac{\hbar^2}{\mu e^2} = \frac{\hbar^2}{\beta m_e e^2} = \frac{a_B}{\beta} \quad (20)$$

wherein  $\beta$  denotes the electron mass enhancement due to quickly oscillating electric and magnetic fields. The classical Debye screening Eq.(19) is valid only in the regime[7]

$$k_B T \gg \frac{e^2}{L} \gg \frac{e^2}{\Lambda_D} \gg \frac{e^2 a}{L^2}, \quad (21)$$

wherein the last inequality on the right hand side of Eq.(21) requires that quantum corrections to Debye screening theory can be neglected. Debye screening itself is a purely classical effect. In the opposite regime that describes the quantum degeneracy, Thomas-Fermi screening is required. In all cases the central theorem Eq.(17) for screening lengths holds true. Finally, the Debye theory for the thermal velocity  $u_T$  in Eq.(9) yields

$$\begin{aligned} \frac{1}{m} &= \frac{\sum_a z_a^2 n_a / m_a}{\sum_a z_a^2 n_a}, \\ u_T &= \sqrt{\frac{k_B T}{m}}, \end{aligned} \quad (22)$$

that again indicates the classical nature of the Debye screening theory.

### III. ELECTRON CORRELATIONS

Consider the following electron capture process in a nucleus in the vacuum,

$$e^- + {}^A_Z X \rightarrow \nu_e + {}^A_{Z-1} X. \quad (23)$$

One can employ a complex scattering length  $f = \tilde{a} + i\tilde{b}$  in the center of inertia reference frame to describe Eq.(23)

TABLE I: Numerical Values of Length Scales

$\beta$	20
$T$	$5.0 \times 10^3$ °K
$a$	$2.65 \times 10^{-10}$ cm
$l_T$	$3.34 \times 10^{-7}$ cm
$L$	$5.0 \times 10^{-5}$ cm
$\Lambda_D$	$1.73 \times 10^{-4}$ cm
$\lambda_T$	$2.36 \times 10^{-8}$ cm

Order of magnitude estimates are given for a chemical cell water plasma. Eqs.(21) and (34) for Debye theory validity holds true.

in the low relative velocity limit; The elastic and total cross sections are thereby

$$\sigma_{el} = 4\pi|f|^2 = 4\pi(\tilde{a}^2 + b^2),$$

$$\sigma_{tot}(v) = \left(\frac{4\pi}{k}\right) \Im m f = \frac{4\pi\hbar b}{\mu v} \quad (v \rightarrow 0). \quad (24)$$

When the nuclei is embedded in a condensed matter plasma, the transition rate per nucleus obeys

$$\Gamma = \lim_{v \rightarrow 0} [v\sigma_{tot}(v)] \bar{n} = \left(\frac{4\pi\hbar b}{\mu}\right) \bar{n}, \quad (25)$$

wherein  $\bar{n}$  is the density of electrons at positions  $\{\mathbf{r}_j\}$  that reside right on top of the nucleus at position  $\mathbf{R}$ ; It is the electron nuclear correlation function

$$\bar{n} = \left\langle \sum_j \delta(\mathbf{R} - \mathbf{r}_j) \right\rangle \quad (26)$$

For the case of a heavy electron  $\tilde{e}^-$  dressed in a cloud of photons scattering off a proton producing a neutrino and a neutron

$$\tilde{e}^- + p^+ \rightarrow \nu_e + n \quad (27)$$

we wish to evaluate the correlation function in Eq.(26) in order to describe the effects of the plasma on the rate for Eq.(27).

#### A. Numerical Length Scales

There are many length scales that need to be considered in the evaluation of the electron correlation function Eq.(26) for the reaction Eq.(27). (i) The effective Bohr radius is given by

$$a = \frac{\hbar^2}{\mu e^2} = \frac{a_B}{\beta} \quad (\text{Effective Bohr Radius}). \quad (28)$$

(ii) The Landau length is

$$l_T = \frac{e^2}{k_B T} \quad (\text{Landau Length}). \quad (29)$$

Let  $n$  represents the number density of electrons which is the same as the number of protons in the neutral plasma. (iii) The mean particle spacing is given by

$$L = n^{-1/3} \quad (\text{Mean Particle Spacing}). \quad (30)$$

(iv) The Debye screening length is

$$\Lambda_D = L \sqrt{\frac{L}{4\pi l_T}} \quad (\text{Debye Screening Length}). \quad (31)$$

(v) The thermal De Broglie wave length is

$$\lambda_T = \sqrt{\frac{2\pi\hbar^2}{\mu k_B T}} \quad (\text{De Broglie Wave Length}). \quad (32)$$

Equivalently, the thermal De Broglie wave length is determined by

$$\int e^{-p^2/2\mu k_B T} \left[ \frac{d^3 \mathbf{p}}{(2\pi\hbar)^3} \right] = \frac{1}{\lambda_T^3},$$

$$\lambda_T = \sqrt{\frac{2\pi\hbar^2}{\mu k_B T}} = \sqrt{2\pi a l_T}. \quad (33)$$

A list of order of magnitude estimates for the above length scales in a chemical cell exhibiting nuclear transmutations is given in TABLE I. Eq.(21) for the validity of Debye theory may be expressed as

$$l_T \ll L \ll \Lambda_D \ll L^2/a, \quad (34)$$

in agreement with the estimates in TABLE I. On the other Maiani *et. al.*[12] has erroneously applied the Debye screening formula in regimes wherein Eq.(34) is violated by a large margin.

#### B. Application to Electron Capture

In the limit of a small thermal De Broglie wave length

$$\lambda_T \ll \Lambda_D,$$

$$\bar{n} \approx n \left\langle \frac{2\pi e^2}{\hbar v} \right\rangle_T \approx \sqrt{\frac{8\pi\mu}{k_B T}} \left( \frac{e^2 n}{\hbar} \right),$$

$$\bar{n} \approx 2 \left( \frac{l_T}{\lambda_T} \right) n \approx \left( \frac{1}{2\pi \lambda_T \Lambda_D^2} \right). \quad (35)$$

Eqs.(35) is a hot Debye screening plasma result equivalent to our previous calculations[10, 11].

#### IV. S-STATE WAVEFUNCTIONS

Employing the definitions

$$E = \frac{\hbar^2 k^2}{2\mu} \quad \text{and} \quad \phi(r) = \frac{2\mu \mathcal{U}(r)}{\hbar^2}, \quad (36)$$

the s-wave radial wave function

$$\psi(r) = \frac{u(r)}{r}, \quad (37)$$

is governed by the potential scattering Eq.(2),

$$u''(r, k) + (k^2 - \phi(r))u(r, k) = 0. \quad (38)$$

Calogero[13] defines a variable phase  $\eta(r, k)$  and variable amplitude  $w(r, k)$  defined by

$$\begin{aligned} u(r, k) &= w(r, k) \sin(kr + \eta(r, k)), \\ u'(r, k) &= kw(r, k) \cos(kr + \eta(r, k)). \end{aligned} \quad (39)$$

The s-wave phase shift  $\eta_s(k)$  is computed in virtue of the limits

$$\begin{aligned} \lim_{r \rightarrow 0} \eta(r, k) &= 0, \\ \lim_{r \rightarrow \infty} \eta(r, k) &= \eta_s(k). \end{aligned} \quad (40)$$

The second order differential Eq.(38) is thereby replaced by two first order differential equations

$$\eta'(r, k) = -\left(\frac{\phi(r)}{k}\right) \sin^2(kr + \eta(r, k)), \quad (41)$$

and

$$w'(r, k) = \left(\frac{\phi(r)}{2k}\right) w(r, k) \sin(2kr + 2\eta(r, k)). \quad (42)$$

Eqs.(40) and (41) determine the variable phase shift which in turn determines the variable amplitude in virtue of Eq.(42); i.e.

$$\begin{aligned} w(r, k) &= w(0, k) \times \\ \exp \left[ \int_0^r \left( \frac{\phi(r')}{2k} \right) \sin(2kr' + 2\eta(r', k)) dr' \right]. \end{aligned} \quad (43)$$

Of interest in what follows is the limit  $k \rightarrow 0$ . This small energy regime is described by the variable scattering length.

#### A. Variable Scattering Length

The scattering length  $\mathcal{L}_s$  is here defined in terms of the s-wave phase shift as

$$\mathcal{F}_s = \lim_{k \rightarrow 0} \left( \frac{\eta_s(k)}{k} \right). \quad (44)$$

The variable scattering length is defined in terms of the variable phase according to

$$\mathcal{F}(r) = \lim_{k \rightarrow 0} \left( \frac{\eta(r, k)}{k} \right). \quad (45)$$

so that

$$\mathcal{F}_s = \lim_{r \rightarrow \infty} \mathcal{F}(r). \quad (46)$$

In virtue of Eqs.(41) and (45), one finds

$$\begin{aligned} \mathcal{F}(0) &= 0, \\ \mathcal{F}'(r) &= -\phi(r)(r + \mathcal{F}(r))^2, \\ \mathcal{F}_s &= -\int_0^\infty \phi(r)(r + \mathcal{F}(r))^2 dr. \end{aligned} \quad (47)$$

The s-wave cross section is thereby

$$\lim_{k \rightarrow 0} \sigma_s(k) = 4\pi |\mathcal{F}_s|^2. \quad (48)$$

Finally, the relative probability, i.e. relative amplitude squared, for the scattering particles to be on top of one another compared with being widely separated

$$\begin{aligned} \lim_{k \rightarrow 0} \left| \frac{w(0, k)}{w(\infty, k)} \right|^2 = \\ \exp \left( -2 \int_0^\infty \phi(r)(r + \mathcal{F}(r)) dr \right). \end{aligned} \quad (49)$$

Eqs.(47) and (49) is central for predicting electron capture rates in the degenerate Thomas-Fermi screening regime.

#### B. Degenerate Electron Capture

Suppose we consider an attractive screened Coulomb potential of the form

$$\mathcal{U}(r) = -\left[ \frac{|Z_1 Z_2| e^2}{r} \right] e^{-(r/\Lambda)}, \quad (50)$$

wherein the screening length  $\Lambda$  is not required to be a Debye screening length. Then  $\phi(r)$  has the form

$$\phi(r) = -\left[ \frac{2}{ar} \right] e^{-(r/\Lambda)}. \quad (51)$$

Eqs.(47) and (51) imply

$$\mathcal{F}'(r) = \left[ \frac{2}{ar} \right] e^{-(r/\Lambda)} (r + \mathcal{F}(r))^2 \Rightarrow \mathcal{F}(r) \geq 0. \quad (52)$$

The scattering length thereby obeys

$$\begin{aligned} \mathcal{F}_s &= \frac{2}{a} \int_0^\infty \frac{1}{r} (r + \mathcal{F}(r))^2 e^{-(r/\Lambda)} dr, \\ \mathcal{F}_s &\geq \frac{2}{a} \int_0^\infty r e^{-(r/\Lambda)} dr = \frac{2\Lambda^2}{a}. \end{aligned} \quad (53)$$

The cross section is bounded from below by

$$\lim_{k \rightarrow 0} \sigma_s(k) = 4\pi |\mathcal{F}_s|^2 \geq \left( \frac{16\pi\Lambda^4}{a^2} \right). \quad (54)$$

As the screening length grows ever larger  $\Lambda \rightarrow \infty$  the scattering cross section also diverges; i.e. the total cross section of an unscreened Coulomb potential is infinite.

The rate of electron capture for this model is determined by Eqs.(25) and (26) via

$$\bar{n} = n \lim_{k \rightarrow 0} \left| \frac{w(0, k)}{w(\infty, k)} \right|^2, \\ \bar{n} = n \exp \left( -2 \int_0^\infty \phi(r) (r + \mathcal{F}(r)) dr \right), \quad (55)$$

in virtue of Eq.(49). From Eqs.(51), (52) and (55),

$$\bar{n} \geq n \exp \left( \frac{4}{a} \int_0^\infty e^{-(r/\Lambda)} dr \right), \\ \bar{n} \geq n \exp \left( \frac{4\Lambda}{a} \right). \quad (56)$$

The lower bound in Eq.(56) is exponentially larger than the prediction of Maiani *et. al.*[12] for the quantum degenerate density  $n$  discussed in this section.

## V. CONCLUSION

Properties of a fully ionized water plasma have been discussed. The theory of the screening of the Coulomb law has been rigorously derived from a thermodynamic viewpoint. A kinetic model was reviewed determining the transition rate per unit time for electron capture by a nucleus and the resulting nuclear transmutations. Corrections to the recent Maiani *et.al.* calculations have been discussed. The regime of validity for Debye screening length has been derived. The WKB approximation erroneously employed by Maiani in the long De Broglie wave was corrected employing the mathematically rigorous Calogero formalism in potential scattering. We have stood by our previous results on the rates of electro electron capture processes for a water plasma in chemical cells.

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